Approximation Algorithms
problem set #1 – due Wednesday, April 19

Problem 1 (Two-Dimensional Bin Packing). Consider the following problem. We are given a collection of items $1, \ldots, n$. Each item i has a weight w_i and size s_i. Our goal is to pack all items in the minimum number of identical bins so that the total weight of items in every bin is at most W, and the total size is at most S.

Give a constant factor approximation algorithm for the problem.

Problem 2 (Max Cut)

a. Give a 2-approximation algorithm for the Max Cut problem. In Max Cut, we are given a graph $G = (V, E)$, and our goal is to partition the set of vertices V into two sets L and R to maximize the size of the cut between L and R. The size of the cut between L and R equals the number of edges with one endpoint in L and the other in R.

b. Give a 4-approximation algorithm for Directed Max Cut. Given a *directed* graph $G = (V, A)$, partition the set of vertices V into two sets L and R to maximize the size of the *directed* cut between L and R. The size of the *directed* cut between L and R equals the number of arcs $(u, v) \in A$ with $u \in L$ and $v \in R$.

Problem 3. Suppose we want to schedule n jobs $1, \ldots, n$ on a single machine. Each job j has a processing time p_j and weight w_j. The completion time C_j of job j is the time when j is completed. Give an exact algorithm for finding the schedule with the minimum weighted completion time defined as follows:

$$weighted\ completed\ time = \sum w_j C_j$$

Problem 4. Give an example of a set of points X in $\mathbb{R}^d$ and an initial set of centers (seed for the algorithm) $c_1, \ldots, c_k$ in $\mathbb{R}^d$ for which Lloyd's algorithm returns a suboptimal solution. What is the approximation of the algorithm in this case?